investigating the characteristics of the propagation modes and the impedance properties of the lines. Straightforward techniques for solving problems have been given and some examples have been worked out for the important case of the three-conductor line.

It should be possible to extend the analysis given here to the case of low-loss lines, where exponential attenuation of the propagating modes will arise, along with continuous conversion of energy from one mode to another. This will be the subject of further investigation.

Acknowledgment

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Excess Losses in *H*-Plane Loaded Waveguides

FRED E. GARDIOL AND OLIVIER PARRIAUX

***Abstract*—The attenuation in a waveguide partially filled with absorbing material can become larger than that of the same waveguide completely filled with that same material. Theoretical and experimental results are presented together with field distributions showing that this excess loss is due to a large concentration of electric field within the lossy dielectric in the partially filled configuration.**

I. Introduction

In a recent publication, Bui and Gagné [1] determined the attenuation in waveguides containing *H*-plane slabs of a lossy dielectric, utilizing a perturbation of the lossless dielectric solution. A most interesting feature of the results presented is that, in several configurations involving high-permittivity dielectrics, larger losses were found in partially loaded waveguides than in completely filled ones. Rather surprised by this unexpected result, the authors suggest that it might be attributed to the approximate nature of the technique used. If true, this would mean that the method and the results presented in [1] are not reliable.

The present study shows that, surprising as they may seem at first, the results obtained in [1] correspond to actual fact and that the attenuation is not necessarily a monotonic function of the filling factor. The "excess" attenuation is caused by the presence of a large concentration of the electric field within the dielectric for the partially loaded waveguide. A similar nonmonotonic behavior appears in results previously published by Arnold and Rosenbaum [2].

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II. Theoretical Results

Since a number of publications have already dealt in some detail with this type of structure [3]–[5], there is no need to repeat the basic theory here. The complex transcendental equation obtained for lossy-dielectric loading can be solved exactly by means of available computer programs [6]. Calculations were made for the longitudinal section magnetic LSM_{11} mode in a waveguide containing a lossy slab next to the broad wall (Fig. 1). Results for the attenuation and phase shift are presented in Figs. 2 and 3 as a function of slab thickness for different conductivities. For conductivities σ much smaller than $\omega\epsilon$, the attenuation curves increase exponentially at first then pass through a maximum in the vicinity of $t/a=0.24$ (for this particular configuration), and finally taper down to the value for the completely filled guide.

For large conductivities [Fig. 2(b)], the attenuation curves behave differently. The attenuation increases sharply for thin slabs (as in the previous case), but the peak of the curve is

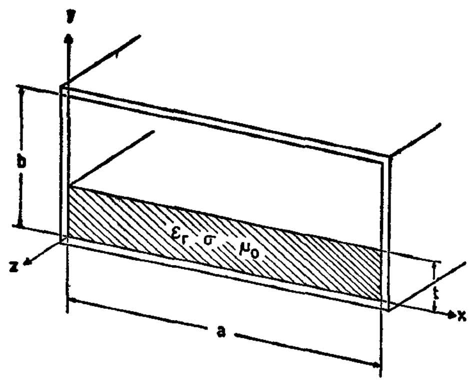

Fig. 1. Rectangular waveguide loaded with an H-plane slab next to the guide wall.

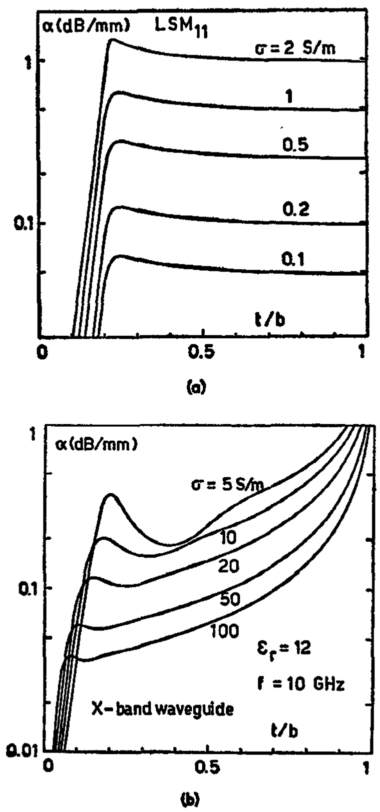

Fig. 2. Attenuation α at 10 GHz as a function of loading (t/b) for an X-band waveguide containing a slab of permittivity $\epsilon_r = 12$ (LSM_{11} mode). (a) $\sigma \lesssim 2S/m$. (b) $\sigma \gtrsim 5S/m$.

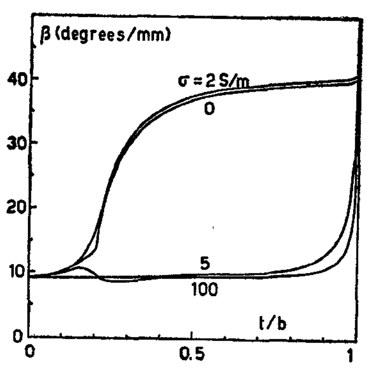

Fig. 3. Phase-shift coefficient β for the same case as Fig. 2 (LSM_{11} mode)

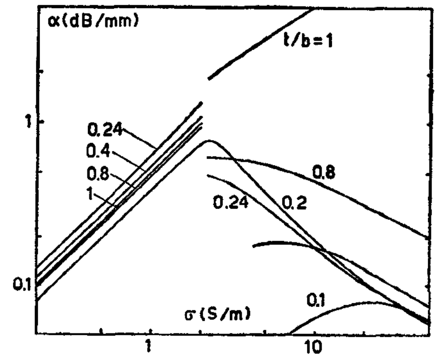

Fig. 4. Attenuation α as a function of conductivity for the case of Fig. 2 (LSM_{11} mode).

shifted to smaller values of t/b and then, after passing through a minimum, the curves increase again and rise very sharply for large filling factors. The same behavior appears in results published in [2]. The phase coefficient β also behaves quite differently at small and large values of σ; up to $\sigma \simeq 2S/m$, β is approximately the same as for lossless dielectrics. A sudden change occurs between 2 and $5S/m$, after that, the value of β does not change appreciably with increasing conductivity, except for t/b very near 1.

The dependence of attenuation on conductivity is shown in Fig. 4 for different slab thicknesses. With small conductivities ($\sigma < 2S/m$ or $\tan \delta < 0.3$), α is proportional to σ. The linear approximation developed in [1] applies over this range. When σ increases further, two quite different phenomena can occur: for a thin slab ($t/b \leq 0.2$), the attenuation curve goes through a maximum and then decreases in a continuous manner, while for larger filling ratios a sudden jump is observed in the vicinity of $t/b = 0.22$. Eventually, the attenuations decrease with increasing σ, with the notable exception of the completely filled waveguide.

The sudden jump observed can be understood by considering the various solutions of the wave equation in the LSM_{1n} set. Previous data considered only the first solution (LSM_{11}) for which the propagation coefficient tends towards the one of the dominant TE_{10} mode in the empty waveguide at the limit $t \rightarrow 0$. However, this does not necessarily mean that this mode will still have the lowest attenuation as the loading is increased. Comparing the LSM_{11} and LSM_{12} attenuations (Fig. 5) for the case $\sigma = 2S/m$, it is clear that the lowest loss corresponds to the LSM_{11} for small filling factors, to the LSM_{12} for larger ones, and, again, to the LSM_{11} for factors close to 1.

A similar effect appears in a waveguide containing E-plane slabs of lossy material [7]. An increase in conductivity introduces a coupling between these two modes in the vicinity of the crossover region, as can be seen by comparing the curve for $\sigma = 3S/m$ (dashed line) with the two curves for $\sigma = 2S/m$ (solid lines) in Fig. 5. The appearance of conductivity-induced coupling explains the sudden jump observed and the two quite different behaviors appearing over different ranges of conductivity.

III. Experimental Results

It was considered most useful at this point to check experimentally the rather surprising results predicted by the electromagnetic theory. Measurements were therefore carried out with slabs of Eccosorb HF-1000 absorbing material

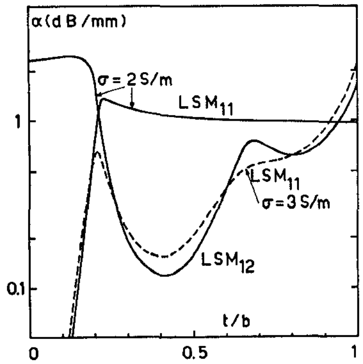

Fig. 5. Comparison of attenuation curves for $\sigma=2S/m$ and $\sigma=3S/m$ (LSM_{11} and LSM_{12} modes).

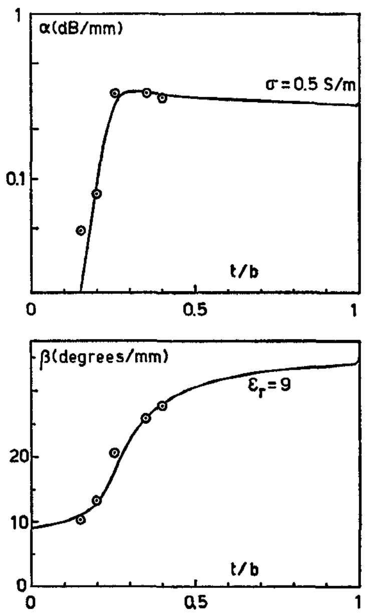

Fig. 6. Comparison of experimental results (dots) with computed values (solid lines) for $\epsilon_r=9$ and $\sigma=0.5S/m$ (Eccosorb HF-1000).

(Emerson and Cuming). Particular precautions were taken to determine accurately the propagation coefficient and avoid errors due to end effects. The scattering-matrix components were determined for two different slab lengths from measurements utilizing the circle or Deschamps' method [8], [9]. The technique used to extract the propagation coefficient from these experimental results is outlined in the Appendix. The scattering matrix of the empty to loaded waveguide transition can be evaluated at the same time (a computer program was written to carry out the complete data processing). The results obtained at 10 GHz are shown in Fig. 6 and agree rather well with the theoretical curve for the LSM_{11} mode with $\epsilon_r=9$ and $\sigma=0.5S/m$.

IV. Electromagnetic Field Distribution

The behavior of the attenuation curves observed in the previous sections is directly linked to the electric-field distribution within the absorbing material. The attenuation can be defined as the half-ratio of the power dissipated over the waveguide cross section to the complete power transmitted through this area [10]. For losses due to a conducting insert, this ratio becomes

$$\alpha = \frac{\int_{\Delta A} \sigma |E|^2 \, da}{2 \,\mathrm{Re} \int_A \bar{a}_z \cdot (\bar{E} \times \bar{H}^*) \, da} \tag{1}$$

where $\bar{a}_z$ is the unit vector along the direction of the propagation, A is the waveguide cross section, ΔA is the insert cross-section, and the asterisk denotes the complex conjugate. If, for a constant power transfer, the electric field is not affected by an increase in insert cross section, the attenuation is proportional to the slab thickness. This condition is approximately satisfied by low-permittivity materials ($\epsilon_r \simeq 1$) [1]. Large permittivity dielectrics modify the electric-field distribution, so that the attenuation becomes a more complicated function of the filling factor.

The electric-field components of LSM_{1n} modes in the loaded waveguide are [4]

$$E_x = -Ae^{-\gamma z}\left(p_d \frac{\pi}{a}\right) \cos\left(\frac{\pi x}{a}\right) \sin(p_d y)$$

$$E_y = Ae^{-\gamma z}\left(\left(\frac{\pi}{a}\right)^2 - \gamma^2\right) \sin\left(\frac{\pi x}{a}\right) \cos(p_d y), \qquad 0 \le y \le t$$

$$E_z = -Ae^{-\gamma z}(p_d \gamma) \sin\left(\frac{\pi x}{a}\right) \sin(p_d y) \tag{2}$$

$$E_x = Be^{-\gamma z}\left(p_a \frac{\pi}{a}\right) \cos\left(\frac{\pi x}{a}\right) \sin\{p_a(b-y)\}$$

$$E_y = Be^{-\gamma z}\left(\left(\frac{\pi}{a}\right)^2 - \gamma^2\right) \sin\left(\frac{\pi x}{a}\right) \cos\{p_a(b-y)\}, \qquad t \le y \le b$$

$$E_z = Be^{-\gamma z}(p_a \gamma) \sin\left(\frac{\pi x}{a}\right) \sin\{p_a(b-y)\} \tag{3}$$

where $p_a = \sqrt{\omega^2\epsilon_0\mu_0 + \gamma^2 - (\pi/a)^2}$, $p_d = \sqrt{\omega^2\epsilon_0\epsilon_r\mu_0 + \gamma^2 - (\pi/a)^2}$, and A and B are amplitude factors. Considering a power transmission of 1 W at $z=0$, they are given by

$$B = A\epsilon_r \frac{\cos p_d t}{\cos p_a(b-t)}$$

$$A = \left(\frac{1}{8} \mathrm{Re}\left(j\omega\epsilon_0\gamma^* \left\{\left(\frac{\pi}{a}\right)^2 - \gamma^2\right\} a \cdot \left\{\epsilon_r^* \left(\frac{\sin 2p_d' t}{p_d'} + \frac{\sinh 2p_d'' t}{p_d''}\right) + |\epsilon_r|^2 \left|\frac{\cos p_d t}{\cos p_a(b-t)}\right|^2 \left(\frac{\sin 2p_a'(b-t)}{p_a'} + \frac{\sinh 2p_a''(b-t)}{p_a''}\right)\right\}\right)\right)^{-1/2} \tag{4}$$

with the prime denoting the real part and the double prime the imaginary part of a complex number.

The amplitudes of the three electric components were calculated for several slab thicknesses and are presented in Fig. 7. It is apparent that the amplitude of all the electric-field

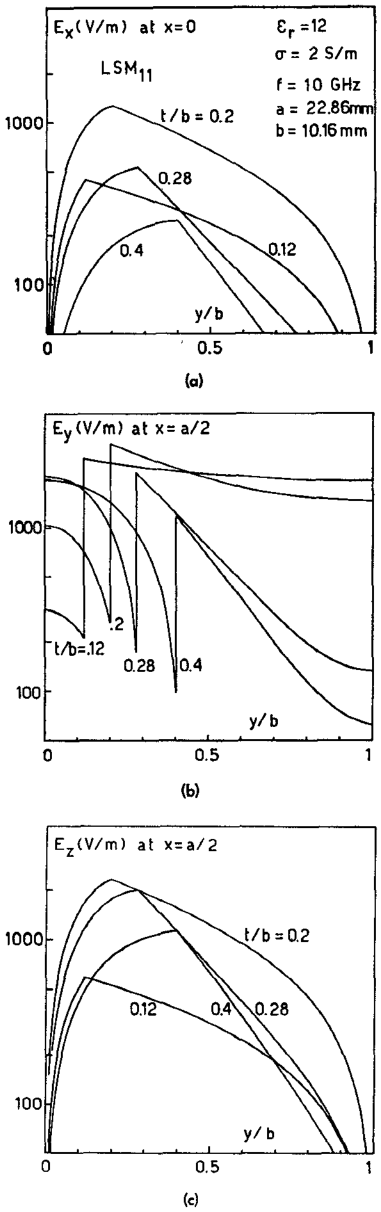

Fig. 7. Amplitude of the electric-field components for a waveguide carrying a power of 1 W. (a) E_x. (b) E_y. (c) E_z (LSM_{11} mode).

components within the lossy dielectric increase with slab thickness when $t/b < 0.2$ and decrease with larger loading. The reduced field concentration explains the observed decrease of the attenuation [see relation (1)].

V. Conclusions

It was shown, both by theoretical derivations and by measurements, that a waveguide can exhibit larger losses when partially filled with an H-plane slab of lossy dielectric than when completely filled with that dielectric. This behavior was shown to be due to the increase in loss within the slab caused by the additional E_x and E_z components which are not present in the completely filled waveguide.

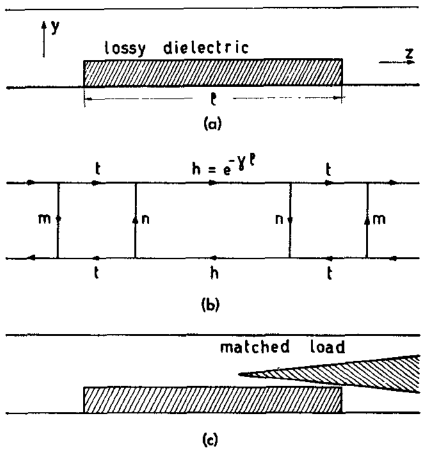

Fig. 8. (a) Loaded section between two empty waveguides. (b) Flowgraph representation. (c) Measurement technique to determine m directly.

Appendix

When only one mode can propagate in both the loaded and the empty waveguide and when the loaded section is long enough to prevent higher order mode interactions between the two ends of the slab, a section of loaded waveguide connected to empty waveguides [Fig. 8(a)] can be represented by the equivalent flow graph of Fig. 8(b). The transmission through the loaded section is defined by the factor $h = e^{-\gamma l}$, while the three terms m, n, and t represent the scattering matrix of the reciprocal transitions from the empty to the loaded waveguide.

Measurements carried out on the loaded section only yield the scattering coefficients for the complete assembly, which is reciprocal and symmetrical:

$$M = S_{11} = S_{22} = m + \frac{h^2 t^2 n}{1 - h^2 n^2}$$

$$T = S_{21} = S_{12} = \frac{h t^2}{1 - h^2 n^2}. \tag{5}$$

This information is not sufficient to determine h (and hence γ), except possibly under particular conditions where some of the terms can be neglected.

By making measurements with different lengths l_1 and l_2 of loaded guide, two sets of results (M_1, T_1, with h_1) and (M_2, T_2, with h_2) are obtained. The two resulting sets of (5) can then be solved, yielding, after some mathematical manipulations

$$m = \frac{(M_1^2 - M_2^2 - T_1^2 + T_2^2) \pm \sqrt{(M_1^2 - M_2^2 - T_1^2 - T_2^2)^2 - 4(M_1 - M_2)\{M_1(T_2^2 - M_2^2) - M_2(T_1^2 - M_1^2)\}}}{2(M_1 - M_2)}. \tag{6}$$

The proper sign for the square root can be determined by carrying out the calculations and determining their meaning. In particular, one must have $|m| \leq 1$. The value of γ can then be obtained from the ratio h_1/h_2:

$$\gamma = \frac{\log_e \left[\left(\frac{M_2 - m}{M_1 - m} \right) \cdot \frac{T_1}{T_2} \right]}{l_1 - l_2}. \tag{7}$$

The remaining terms of the scattering matrix representing the transition from the empty to loaded waveguide can also be determined at this point:

$$n = \frac{M_1 - m}{T_1 e^{-\gamma l_1}} = \frac{M_2 - m}{T_2 e^{-\gamma l_2}} \tag{8}$$

$$t = \sqrt{\frac{T_1(1 - n^2 e^{-2\gamma l_1})}{e^{-\gamma l_1}}} = \sqrt{\frac{T_2(1 - n^2 e^{-2\gamma l_2})}{e^{-\gamma l_2}}}. \tag{9}$$

Some precautions are necessary, in particular, those due to the determination of m. The relation obtained (6) contains several subtractions, both in the numerator and the denominator. This calculation is very sensitive to experimental errors; hence, the utmost care is needed during measurements. One possible way to avoid this delicate step is to determine m directly, which can be done by introducing the tip of a lossy termination inside of the loaded waveguide section [Fig. 8(c)], in which case $h \rightarrow 0$ and $M = m$. Of course, this can only be done with rather thin slabs.

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Analysis of Thick Rectangular Waveguide Windows With Finite Conductivity

RAYMOND J. LUEBBERS AND BENEDIKT A. MUNK

***Abstract*—The modal analysis method is used to calculate the reflection and transmission properties of a thick rectangular window centrally located in a rectangular waveguide. Excellent agreement is obtained between calculated and measured values for windows of intermediate thickness. For thicker windows made of finitely conducting materials, the results obtained using perfectly conducting waveguide modes are inaccurate. However, by modifying the modes so as to include some of the mode-coupling effects caused by the surface currents, good agreement between calculated and measured data is obtained for a very thick, finitely conducting window.**

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INTRODUCTION

IN THIS PAPER we are concerned with calculating the reflection and transmission properties of a thick, finitely conducting rectangular resonant window in a rectangular waveguide. The geometry of the problem and the coordinate system used are shown in Fig. 1. The window is assumed to be centered in the waveguide with the energy propagating in the z direction.

If the slot is infinitesimally thin, i.e., $l \rightarrow 0$, variational techniques can be used to obtain an expression for the equivalent shunt impedance from which the reflection and transmission coefficients can be calculated [1, pp. 88–97]. For finite values of l, however, this method appears to be very difficult to apply, except for the degenerate cases where the slot width equals the waveguide width (capacitive obstacle)